Mathematics Department - Issue 1.7

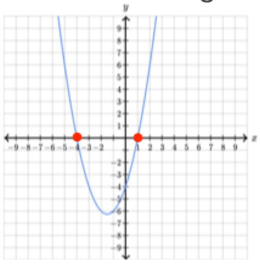
Higher Tier GCSE Mathematics – **Given Formulae** details section 1.

1. Students are **not** expected to know the following formulae included in the subject content; they are given these on the formulae sheet insert in the exam for 2025. **Students should know how to use these formulae**. Students should refer to the formulae sheet insert that they have been given to see how these are presented on this insert.

The quadratic formula

The solutions of quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$ are given by:

$$\boxed{x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}}$$



The formula will give the two solutions to the quadratic equation. These are known as the roots of the equation and are the points where the graph of the quadratic cross the x axis (when $y = 0$).

Circumference and area of a circle

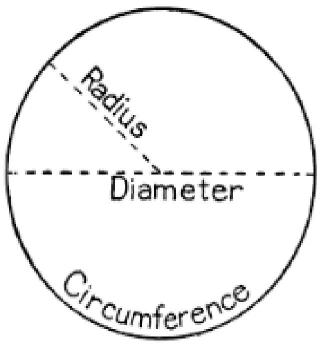
Where *r* is the radius and *d* is the diameter:

Circumference of a circle

$$\boxed{C = \pi d \text{ or } C = 2\pi r \quad (\text{because } d = 2r)}$$

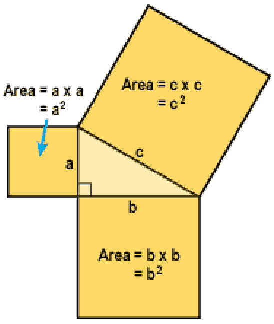
Area of a circle

$$\boxed{A = \pi r^2}$$



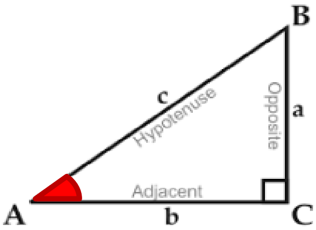
Pythagoras' theorem

This only applies in right angled triangles and used to find a missing side length when the other two side lengths are known. In any right-angled triangle where *a*, *b* and *c* are lengths of the sides and *c* is the hypotenuse (the longest side):



$$\boxed{a^2 + b^2 = c^2}$$

Note: To find the longest side the (hypotenuse), c in the diagram, we 'square square **add** then square root'. To find either of the other two shorter sides we 'square square **subtract** then square root'.



Trigonometry formulae - SOHCAHTOA

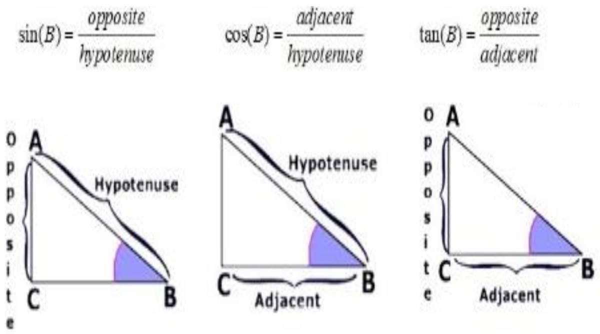
In any **right-angled** triangle *ABC* where *a*, *b* and *c* are lengths of the sides and *c* is the hypotenuse (the longest side).

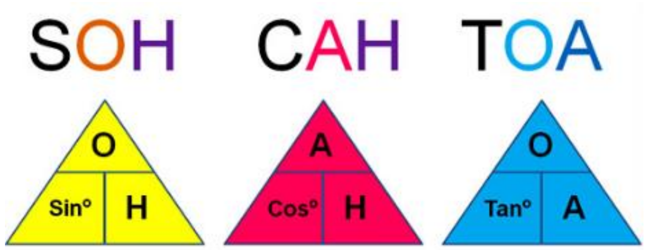
$$sinA = \frac{a\ (opposite)}{c\ (hypotenuse)}\ ,\ cosA = \frac{b\ (adjacent)}{c\ (hypotenuse)}\ ,\ tanA = \frac{a\ (opposite)}{b\ (adjacent)}$$

(A represents the angle at the vertex A which is opposite the side a)

To find a side length select the correct trig ratio sin, cos or tan and re-arrange as necessary.

To find an angle select the correct ratio and the two side lengths and use the the inverse function e.g. inverse Cos function cos^{-1} on a calculator.



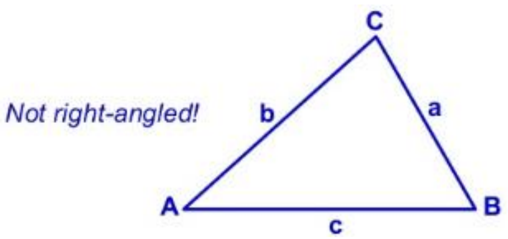


Trigonometry formulae - the sine rule

In **any non-right-angled** triangle *ABC* where *a*, *b* and *c* are lengths of the sides use the **sine rule** or **cosine rule** depending on the information given in the diagram or question.

sine rule: $$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} \quad \text{or} \quad \frac{\sin A}{a} = \frac{SinB}{b} = \frac{sinC}{c}$$

(A represents the angle at the vertex A which is opposite the side a, B represents the angle at the vertex B which is opposite the side b and C represents the angle at the vertex C which is opposite the side c). Use this rule when you need to find an angle and you know the side opposite the angle and you also know another side. Use this rule to find a side when you know the angle opposite the side and also another side and the angle opposite that side.



Note also that the formula is used in pairs and should be re-arranged to find the missing angle/side you require. To find the angle A,B or C the inverse sine function sin^{-1} should be used on a calculator.

Higher Tier GCSE Mathematics – **Given Formulae** details section 1 continued..

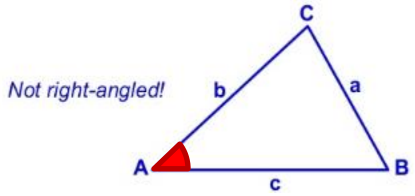
Trigonometry formulae - the cosine rule

Cosine rule for finding a side length: $a^2 = b^2 + c^2 - 2bccosA$

(A represents the angle at the vertex A which is opposite the side a. b and c represent the lengths of the other two sides.) Use this formula to find a side when you know the two other sides and the angle opposite the side you are trying to find.

Student will be expected to be able to re-arrange the cosine rule above to work out an angle given three sides. Here A is the angle opposite side a and $b$and c are the other two sides**.** Use this formula to find an angle when you know all three sides.
This formula will not be given.

Cosine rule for finding an angle $CosA = \frac{b^2 + c^2 - a^2}{2bc}$

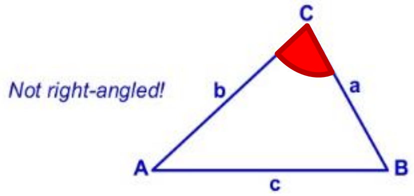


(note that when the value of $CosA$ is calculated a calculator is used to find the value of the angle A using the inverse Cos function Cos^{-1}

Trigonometry formulae – use for finding the area of a triangle

For any triangle where the height is unknown the **area** of the triangle can be found using this formula.

Area of a triangle $= \frac{1}{2}ab\ sinC$

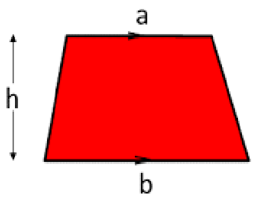


(a and b are two sides of the triangle and C is the angle between them.)
To use this formula you need to know two sides and the angle between those two sides.
Note that where the base and height perpendicular to the base are known the standard formula for the area of half x base x height can be used.

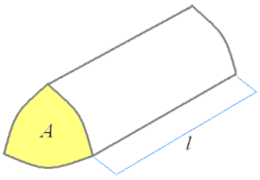
Higher Tier GCSE Mathematics – **Given Formulae** details section 1 continued..

Perimeter, area, surface area and volume formulae

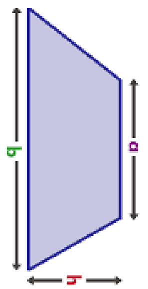
Where *a* and *b* are the lengths of the parallel sides and *h* is the perpendicular distance between these two parallel sides:



Area of a trapezium $= \frac{1}{2}(a+b)h$



Volume of any prism = area of cross section x length



Students should also know how to calculate the perimeter and surface area of common shapes. This will include the surface area of a cylinder.

Compound interest

Where *P* is the principal amount (the amount invested), *r* is the interest rate over a given period and *n* is number of times that the interest is compounded (could be years, months etc)

Total accrued (total amount at the end of the period n) $= P(1 + \frac{r}{100})^n$

(Note that r divided by 100 just changes the interest rate to a decimal value)

Repeated Percentage Change

This formula for compound interest may be applied for repeated percentage changes that are not just compound interest. For example a population may be decreasing by 15% per month for 34 months from an initial population of 40,000. To find the population after 34 months then the formula becomes:-

Final population $= 40000\ X\ (1 - 0.15)^{34}$ (note: 1 – 0.15 = 0.85 a decrease of 15%)

Probability

Where *P*(*A*) is the probability of outcome *A* and *P*(*B*) is the probability of outcome *B* then:

For mutually exclusive events

P(*A* or *B*) = *P*(*A*) + *P*(*B*)
P(*A* and *B*) = *P*(*A*) x *P*(*B*)

For non mutually exclusive events

P(*A* or *B*) = *P*(*A*) + *P*(*B*) - *P*(*A* and *B*)
P(*A* and *B*) = *P*(*A*) x *P*(*B*)

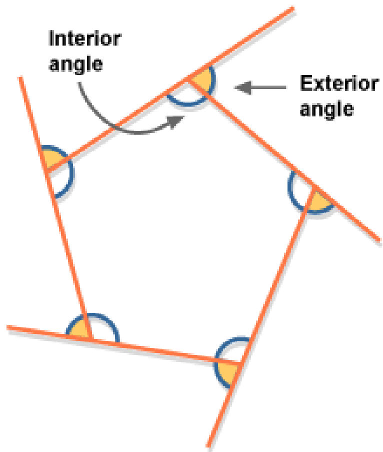
Note - Relative Frequency – This is just another term used for **Probability** calculated using data from real-life or an experiment/test. Quite often this is given as a decimal value.

Higher Tier GCSE Mathematics – Formulae **not given** details section 2.

2. Students **are expected to know the following** formulae or be able to derive them; **they will not be given in the exam.**

Angles in polygons (2D shapes)

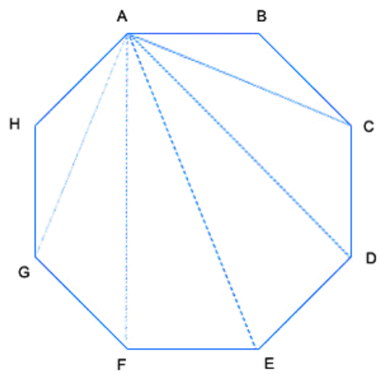
The following three formulae/rules relating to angles in polygons need to be learnt and students need to be able to apply them to solve problems.



The total sum of the interior angles of any polygon

There are altogether (n-2) triangles in a polygon where n is the number of sides of the polygon.

Sum of angles of each triangle = 180°

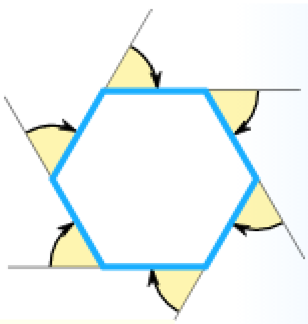


Total sum of the **interior angles** of n-sided polygon

= (n-2) x 180°

where n is the number of sides

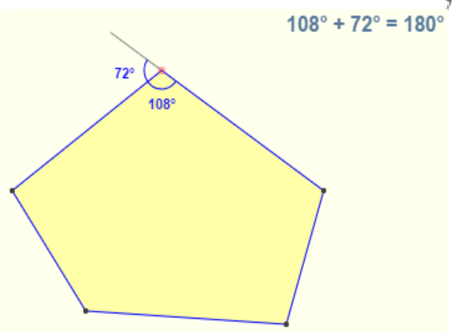
The total sum of the exterior angles of any polygon



Total sum of the **exterior angles** of a polygon

Total of exterior angles = 360°

Sum of an interior and exterior angle of a polygon



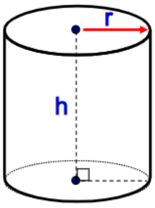
Sum of the **interior and exterior angle** of a polygon

= 180°

(as angles on a straight line = 180°)

Higher Tier GCSE Mathematics – Formulae **not given** details section 2 continued..

Volume and Surface Area of a Cylinder

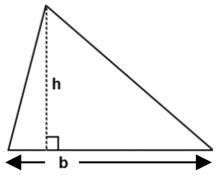


$$\text{Volume} = \pi r^2 h$$

$$\text{Surface Area} = 2\pi r h + 2\pi r^2$$

Area of a Triangle

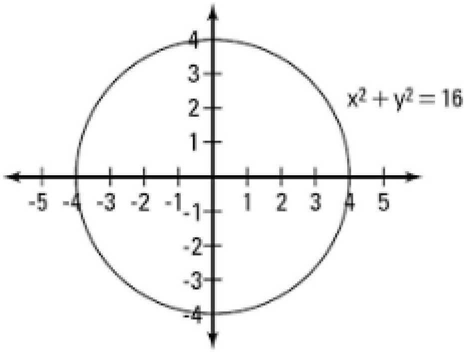
$$\frac{1}{2}\ base\ X\ height\ or\ \frac{b\ X\ h}{2}$$



Equation of a circle

For a circle centre (0,0) on a x-y coordinate grid the equation of the circle is given by:-

$\boldsymbol{x^2 + y^2 = r^2}$ where r is the radius of the circle.



Higher Tier GCSE Mathematics – Formulae details section 3.

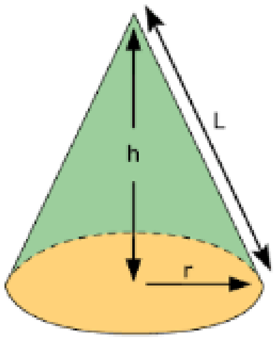
3. Students **are not expected to memorise the following formulae**; **they will be given in the exam with the relevant question**. The diagrams below **may not be given** so students will need to know what each part of each formula represents and be able to use the formula.

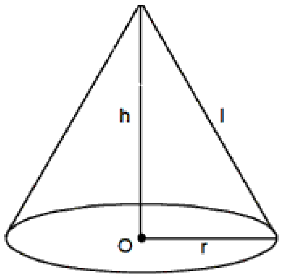
Perimeter, area, surface area and volume formulae

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone or a pyramid:

Curved surface area of a cone $= \pi r l$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

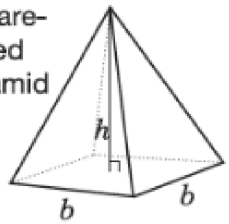




$$l = \sqrt{h^2 + r^2}$$

Volume of any pyramid $= \frac{1}{3}(base\ area)h$

Square-based pyramid

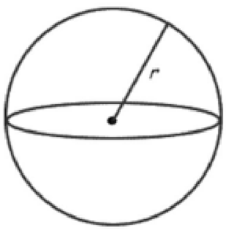


$$V = \frac{(\text{area of base})(\text{height})}{3}$$

$$V = \frac{1}{3} b^2 h \quad \text{or} \quad V = \frac{b^2 h}{3}$$

Surface area of a sphere $= 4\pi r^2$

Volume of a sphere $= \frac{4}{3}\pi r^3$



Higher Tier GCSE Mathematics – Formulae details section 3 continued…

Kinematics formulae (motion)

Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement (distance) from the position when t = 0 and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Higher Tier GCSE Mathematics – Formulae details section 4.

4. Students should be **able to recall** these additional facts as they may be required to solve problems on any of the three papers and **are not likely to be given** in the question.

Trigonometric Ratios (Trig exact values) that need to be learnt

In addition to knowing how to use trigonometry in right angled triangles the following need to be learnt. You should also learn how to derive these using isosceles and equilateral triangles.

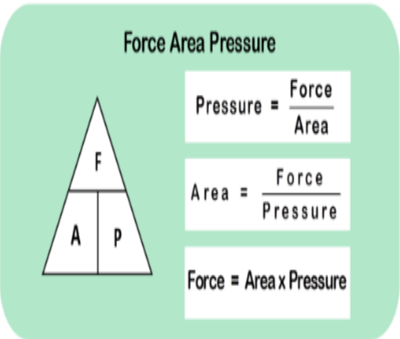
$Sin\ 0° = 0$ $Sin\ 30° = 1/2$ $sin\ 45° = \frac{\sqrt{2}}{2}$ $Sin\ 60° = \frac{\sqrt{3}}{2}$ $Sin\ 90° = 1$

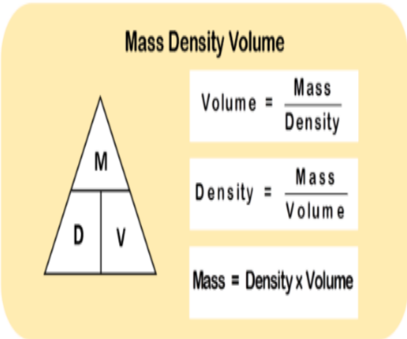
$Cos\ 0° = 1$ $Cos\ 30° = \frac{\sqrt{3}}{2}$ $Cos\ 45° = \frac{\sqrt{2}}{2}$ $Cos\ 60° = 1/2$ $Cos\ 90° = 0$

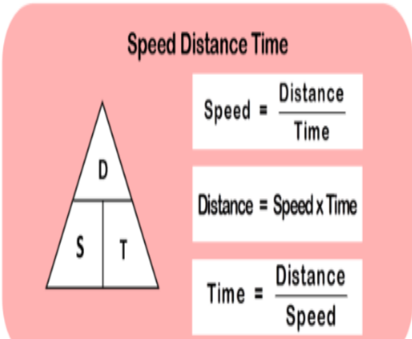
Tan 0° = 0 Tan 30° = $\frac{\sqrt{3}}{3}$ Tan 45° = 1 Tan60° = $\sqrt{3}$

Compound measures that need to be learnt

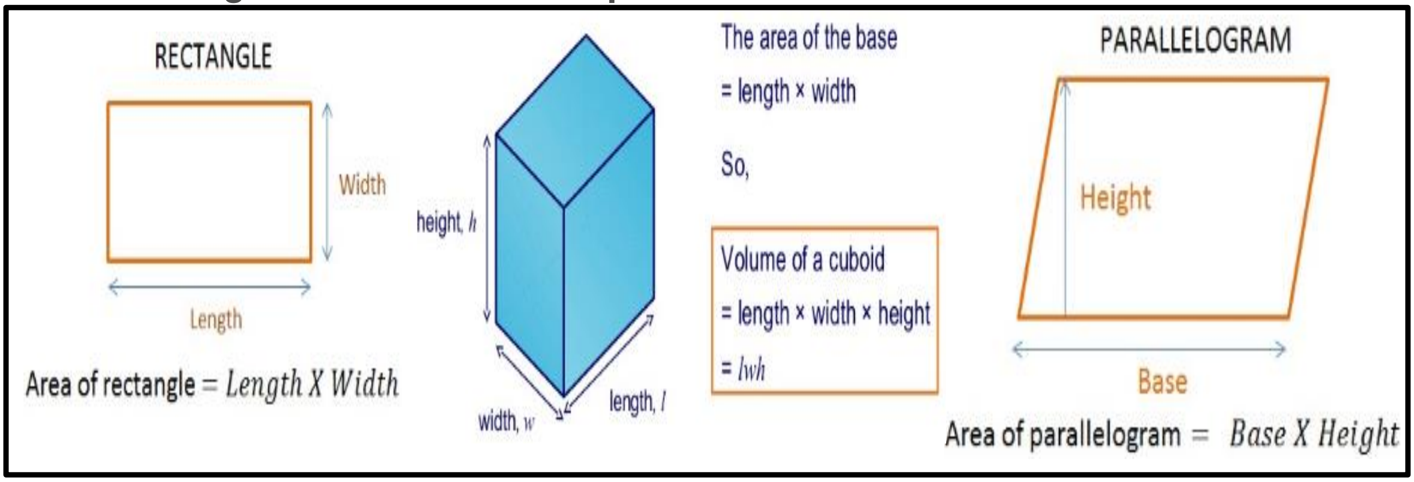
The following compound measures **need to be learnt**:-







Don’t forget to learn these simple area and volume formulae



Conversion facts that should be learnt (they may/may not be given)

5 miles ≈ 8 kilometres so 1 mile ≈ 1.6 kilometres

1 gallon ≈ 4.5 litres (note this is UK gallons)

2.2 lbs = 1 kilogram   1 lb = 0.45kg (lbs is weight in pounds)

1 gallon = 8 pints (note this is UK gallons)

1 inch ≈ 2.5 centimetres

10mm = 1cm, 1000mm = 1m, 100cm = 1m, 1000m = 1km (km = kilometres)

1000mg = 1g, 1000g = 1kg, 1000kg = 1 tonne (mg = milligrams)

1000ml = 1l, 100cl = 1l (ml = millilitres)

1 million is 1000000 (6 zeros)

$1000cm^3$ = 1l (one thousand cubic centimetres equals one litre)

Students should also be able to convert between different units of area and between different units of volume. For example:-

$1m^2 = 1 \; X \; 100 \; X \; 100 \; cm^2 = 10000cm^2$

$1m^2 = 1 \; X \; 1000 \; X \; 1000 \; mm^2 = 1000000mm^2$

$50cm^2 = 50 \div 100 \div 100 \; m^2 = 0.005m^2$

$5.2km^2 = 5.2 \; X \; 1000 \; X \; 1000m^2 = 5200000m^3$

$1m^3 = 1 \; X \; 100 \; X \; 100 \; X \; 100 \; cm^3 = 1000000cm^3$

$1mm^3 = 1 \div 10 \div 10 \div 10 \; cm^3 = 0.001cm^3$

$356900cm^3 = 356900 \div 100 \div 100 \div 100 \; m^3 = 0.3569m^3$

$6.43m^3 = 1 \; X \; 1000 \; X \; 1000 \; X \; 1000 \; mm^3 = 6430000000mm^3$